

Reflections on the nature of analysis and some analytical skills

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Abstract

Before a meaningful account can be given of scholarly communication an investigation of the nature of *analytical skills* is required – the aim of this article. It sets out to come to terms with the meaning of *analysis* by showing that it rests on two mutually cohering features, namely identifying and distinguishing. Since identification implies that multiple features are united and since these traits are not only universal but ultimately indefinable, it is argued that synthesis (identification) is not opposed to analysis but to the other ‘leg’ of analysis, namely distinguishing. Since identification is nothing but the capacity we have to form concepts, the nature of concepts is explained in more detail. Moreover, analysis is not simply constituted by acts of dividing (setting apart), although it cannot reveal its meaning apart from its coherence with the numerical awareness of the one and the many and the original spatial meaning of a whole with its parts. Since the discipline of mereology, as a study of the whole-parts relation, involves modern (mathematical) set theory it is argued that sets are not purely arithmetical for they are, as Kurt Gödel said, *quasi-spatial*. Acknowledging the uniqueness of number and space alongside their mutual coherence serves to illustrate what a non-reductionist ontology means. This digression opens up the possibility to illustrate the nature of analytical skills by arguing that the arithmeticistic claims of modern set theory begs the question (it assumes what it wants to conclude) owing to the underlying antinomy present in its attempt to explain spatial continuity exclusively in numerical terms. This is done on the basis of briefly highlighting conflicting schools of thought within a number of natural sciences and humanities (by and large instantiating ismic orientations opting for a *monistic* perspective). Explaining briefly the equally intrinsically antinomic nature of modern historicism prompted the plea for a non-reductionist ontology as one of the most important guidelines for the enhancement of critical analytical skills. A brief reference is made to the requirement of critical solidarity, immanent critique, factual critique and the importance of critically unveiling the theoretical paradigm of a thinker as well as the ultimate commitment directing the thought of such a thinker.

ORIENTATION¹

Western civilization increasingly became obsessed with the acquisition of so-called *critical skills*, understood in the sense of developed capacities or competencies enabling people to play a significant and reliable role within the highly differentiated

scene of modern societies. The *Higher Education Qualifications Framework*² in particular mentions *critical skills* as an outcome of academic studies on the M. and D. level. This article aims in a twofold way at an explicit delimitation of the importance of critical analytical skills. First of all skills are qualified by the adjective *analytical* and in the second place they are limited to *scholars only*. Of course this delimitation does not mean that only scholars, involved in academic contemplation, are capable of exercising analytical abilities, for within the context of everyday life every human being is constantly involved in *analytical* activities.

Contemplating the enhancement of the analytical skills of scholars immediately invites an analysis of the meaning of analysis! Does this imply that we will end up in a vicious circle? Is it possible to analyze what analysis is all about without presupposing an insight into the nature of analysis? We want to argue that an investigation of the nature of analysis will enable us to discern the fundamental (onto-)logical principles guiding critical thinking and it will unveil the limits of analysis and concept formation as such because the acknowledgement of indefinable (primitive) terms ultimately reflects an awareness of what is unique and nonetheless intimately coheres with everything else.

Primitive terms are inevitable

The moment it is attempted to understand what analysis entails a general consideration surfaces. The issue is the use of *explanatory terms*, for whatever terms we may use in order to *define* analysis each one of those terms may call for a further definition, once again using other terms. Is it possible to continue such an introduction of new terms indefinitely? The (im-)possibility of endlessly introducing new terms is known as a *regressus in infinitum*. In order to avoid this impasse the only alternative is to realize that ultimately every definition terminates in the employment of *undefined* terms, also known as *primitive* terms.

Analysis and synthesis

The next question is whether or not there is a way in which one can systematically identify what is truly primitive and indefinable? A possible candidate is to look at the *etymology* of words. In the case of the term *analysis* such an etymological investigation normally starts with the Greek origins of Western reflection – a legacy in which analysis is considered in intimate connection with *synthesis*. The Greek term *ἀνάλυσις* initially meant *resolution* and *reduction*, while its opposite, *synthesis* (*σύνθεσις*) designates *composition* and also *addition* (see Oeing-Hahnoff 1971, 232). After these terms entered Greek philosophy Zeller explains that in the philosophy of Aristotle analysis means tracing what is given back to the building blocks composing it and through which it came into being (Zeller 1963-II/2, 186).

The connection between separation (dividing = analysis) and recombination (synthesis) implicitly represents the two sides of the relation between a whole and its parts – the theme of the next sub-paragraph.

The whole-parts relation

The juxtapositioning of analysis and synthesis makes an appeal to our awareness of a *whole* with its (constituting) *parts*. During the 20th century there even emerged a systematic discipline focused on the *whole-parts relation*, known as *mereology*. St. Leńiewski, elaborating some ideas of Twardowski and exploring what Husserl introduced in his ‘Logische Untersuchungen’ (*Logical Investigations*) sets out to analyze the relation between a whole and its parts. He coined the term *mereology* and was the first to subject this relation to a formal (*set theoretical*) analysis. Its intention is to embrace both the *element-set* relation and the *set-subset* relation. The Aristotelian-Scholastic legacy is reflected in the distinction between *distributive totalities* (related to universality) and *collective* (integrating) *totalities* (cf. Lorenz 1980, 1145–1146) – a distinction that clearly reflects the two sides of the whole-parts relation.

In an article in which he highlights fascinating similarities between Aristotle’s views and the modern discipline of space (topology), White nonetheless remarks that Aristotle and (modern mathematical) topology part ways in respect of ‘the collective identification of sets of points with (proto)topological regions: topology accepts such an identification as fundamental while Aristotle will have none of it’ (White 1988, 7). The jump from (atomistic) points to continuity then emerges: ‘However, topology maintains that when sufficiently large classes of points are considered collectively, these can, in some cases, be identified with spatial regions in the intuitive proto-topological sense. The result is that, from the topological perspective, continuity becomes an “emergent” property’ (White 1988, 8). This remark is indebted to the arithmeticistic inclination of (set theoretical) mathematics – we shall return to the antinomic character of arithmeticism below. In the current context we only point out that sets cannot be understood in a purely arithmetical way.

Remark: The term *holistic*

It is currently fashionable to defend a so-called ‘holistic’ perspective. Most scholars using this term are not aware of the fact that it is just a term serving to designate the opposite of *atomism*. Within the context of reflections on human society the opposition of atomism and holism is known as that between individualism and universalism. Whereas atomism over-emphasizes the discreteness of number (or analogies of number with other cosmic aspects), holism over-emphasizes the spatial whole-parts relation or its analogies within other aspects. However, most people employing the term ‘holistic’ actually merely want to advocate of perspective aimed at avoiding a limited approach merely focusing one or a few aspects of an issue. They intend to account for *all* the facets at stake. The term ‘holistic’ could be avoided simply by referring to a multi-aspectual nature of the issue under consideration.

Sets are not purely arithmetical

The escape route of introducing *continuity* as an *emergent* property hides a more fundamental shortcoming. Set theory appears to be a purely *arithmetical theory*, dealing with different kinds of multiplicities, called *sets*. Yet, looking at Cantor's initial understanding of a set there are at least *two* features present in his idea of a set: (i) a multiplicity of (conceptually) distinct elements and (ii) the combination of these distinct elements into a completed whole (see Cantor 1962, 282; 379; 387; 411).

Multiplicity and *wholeness* are therefore constitutive for Cantor's notion of a set. Is it possible to understand this relationship purely in numerical (quantitative) terms? On the one hand one cannot deny that the concept of a set has its foundation in a quantitative multiplicity – the one and the many. Yet the numerical aspect of reality is insufficient to account for the nature of the whole-parts relation. What is needed is to consider the nature of *spatial continuity* as well. The most prominent recognition of the *spatial home* of the terms *wholeness* and *totality* is found in the thought of the mathematician Paul Bernays, the co-worker of the foremost mathematician of the 20th century (David Hilbert). He states that it is recommendable not to distinguish the arithmetical and geometrical intuition according to the moments of the spatial and the temporal, but rather by focusing on the difference between the *discrete* and the *continuous* (Bernays 1976, 81). Being fully aware of the claims made by modern mathematics, namely that it is possible to *arithmetize* this discipline fully, it is all the more significant to know that Bernays questions the attainability of this ideal of a complete arithmetization of mathematics – and he does this on the basis of acknowledging the original spatial meaning of continuity and wholeness (totality). He unambiguously writes:

We have to concede that the classical foundation of the theory of real numbers by Cantor and Dedekind does not constitute a complete arithmetization of mathematics. It is anyway very doubtful whether a complete arithmetization of the idea of the continuum could be fully justified. The idea of the continuum is after all originally a geometric idea (Bernays 1976, 187–188).³

Whereas our basic awareness (intuition) of number cannot side-step *succession*, the meaning of space by contrast is determined by simultaneity. The idea of a set combines both these elements, *multiplicity* (succession) and *at once*.

Multiplicity and simultaneity

Particularly in explaining the difference between what is traditionally known as the potential and the actual infinite, the difference between *succession* and *at once*, as well as the irreducibility of the notion of a *totality* surfaces. The extension of any spatial figure is bound to a specific dimension and also to the spatial order of simultaneity, for if all the parts (i.e. a multiplicity of them – evincing the undeniable *quantitative* foundation of space) of such a figure are not present *at once*, the spatial figure concerned is also absent. Only when all three sides of a triangle are

simultaneously present do we have a complete triangle – the succession of its three sides does not yet constitute a triangle.

Space presupposes number

How else will it be possible to understand the meaning of one or more dimensions (such as length, width and height) or different magnitudes (such as 9 centimeters, 5 square meters or 3 liters)? Our awareness of spatial extension is always related to an understanding of number, for extension is always extension in some *dimension* – such as that of a straight line (one-dimensional extension), that of a surface (two-dimensional), etcetera – and it is at once attached to an awareness of the *connectedness* of whatever is extended in a spatial sense. Moreover, what is connected *hangs together*, that is, *coheres*, and this implies that coherence embraces *every connected part*. When every part is given, it is understood as a continuous whole.

This shows that the relation between a *whole* and its *parts* originally belongs to the core meaning of space. But if it belongs to the core meaning of space, then continuous extension is indefinable and can only be replaced by *synonyms*. Interchanging ‘continuity’ with terms like ‘uninterrupted’, ‘connected’, ‘coherent’, and so on, simply repeating what is meant with the term *continuity*, instead of defining it. For this reason Shapiro concedes that the feature of being coherent cannot be characterized in a noncircular way: ‘coherence is not a rigorously defined mathematical concept, and there is no noncircular way to characterize it’ (Shapiro 1997, 13).

Without entering into a more extensive investigation of the interconnections between number and space, at least one outstanding feature of the spatial aspect ought to be mentioned, namely the fact that *continuity* allows for an endless (sub) division – immediately reminding us of the one element traditionally attached to the meaning of *analysis* (sub-dividing). Aristotle, in following up certain insights of Anaxagoras, holds it to be self-evident that ‘everything continuous is divisible into divisible parts which are infinitely divisible’ (*Physica* 231 b 15 ff.). Already the way in which Parmenides has characterized being illuminates important features of continuity and the whole-parts relation. The B Fragments 2 and 3 of Parmenides, contained in Diels-Kranz (1959–1960), hold that being ‘... was not and will never be because it is connected in the present as an indivisible whole, unified, coherent’ (B Fragment 8, 3–6).

The reaction of intuitionistic mathematics to atomism

Modern intuitionistic mathematics made an appeal to these insights of Greek thinking in developing their alternative to the *atomism* entailed in the thought of Cantor, and the formalism of Hilbert.⁴ The intuitionist Hermann Weyl, for example, points out that the fact that it ‘... has parts, is a basic property of the continuum’, and adds: ‘... it belongs to the very essence of the continuum that every one of its parts admits a limitless divisibility’ (Weyl 1921, 77).

These considerations make it clear that White did not realize that what he labeled as the *collective* already entails the wholeness of spatial continuity and that therefore the attempt to see continuity as *emerging* from a collective totality begs the question (i.e. what is supposed to emerge is pre-supposed). A similar problem hampers the above-mentioned discipline of mereology, for it did not enter into an analysis of the whole-parts from the perspective of the uniqueness and mutual coherence between the aspects of number and space. Had it done that it would have realized that set theory itself is dependent upon a specific (though *implicit*) view of the interrelatedness of number and space. When a set is defined as the collection of properly distinct (*wohlunterschiedenen*) things/entities (*Objekten*) that are bound together into a whole (*Ganzen*),⁵ then this circumscription at once made an appeal both to *number* and to *space*. It is therefore not surprising that Wang mentions that Gödel ‘speaks of sets as being “quasi-spatial”’ and then he adds the remark: ‘I am not sure if he would say the same thing of numbers’ (Wang 1988, 202). Clearly, sets are not purely numerical – they ‘borrow’ the whole-parts relation from space. (Just recall the statement of Bernays who said that the geometrical idea of the continuum is expressed by analysis in the language of arithmetic.)

Does analysis merely entail a division?

Regarding the nature of *analysis* we are now justified in establishing that its meaning comes to expression in its coherence *both* with a numerical multiplicity and the divisibility of a spatial whole. However, when already Proclus emphasizes the method of ‘dihairesis’ (subdividing) he merely focuses on the *divisibility* of what is given as a *whole*. This understanding calls for its counter part, namely *synthesis* through which the analyzed parts could be re-united into a *whole*.

Yet the meaning of analysis does not merely reflect its coherence with the divisibility element of the spatial whole-parts relation, for it also comes to expression through the interconnectedness between analysis and number. The distinctness of any two numbers underlies the analytical possibility to *identify* and to *distinguish* them.⁶ For that reason one should rather speak of the logical-analytical aspect. Furthermore, identifying and distinguishing are normed by the logical principles of *identity* and *non-contradiction*⁷ and whenever the analytical acts of identifying and distinguishing derail one ends up with *illogical* concept formation.

Inter-aspectual coherence

Every attempt to deduce the meaning of number from the meaning of analysis (or: logic) is faced with a *vicious circle*. Cassirer is quite explicit in this regard for he claims that a critical analysis of knowledge, in order to side-step a *regressus in infinitum*, has to accept certain basic functions which are not capable of being ‘deduced’ and which are not in need of a deduction (Cassirer 1957, 73–74). David Hilbert also points at this ‘catch 22’ entailed in the logicist attempt to deduce the

meaning of number from that of the logical-analytical mode. In his *Gesammelte Abhandlungen* Hilbert writes:

Only when we analyze attentively do we realize that in presenting the laws of logic we already had to employ certain arithmetical basic concepts, for example the concept of a set and partially also the concept of number, particularly as cardinal number [*Anzahl*]. Here we end up in a vicious circle and in order to avoid paradoxes it is necessary to come to a partially simultaneous development of the laws of logic and arithmetic (Hilbert 1970, 199).⁸

Analysis and concept-formation

At this point we may move ahead and investigate the two ‘legs’ of analysis, namely identifying and distinguishing (see Figure 1). On the one hand it is striking that *abstraction* is normally described in terms of acts of *lifting out* (what is shared) and *disregarding* (what is different) – and *lifting out* matches identifying while *disregarding* matches distinguishing. On the other identifying is nothing but acquiring a concept of something.

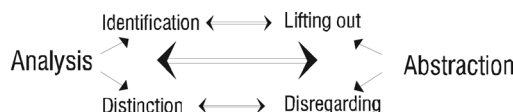


Figure 1:

What is a concept?

Although the term *concept* is frequently used by scholars in all disciplines, most of them do not have a concept of a concept. A concept is subject to logical principles, such as the mentioned principles of identity and non-contradiction. For this reason a concept can either conform or it can violate logical principles. The concept of a *triangle* (as defined below) is logically sound, whereas the concept of a ‘square circle’ is illogical, since it violates the logical principles of identity (‘a circle is a circle’) and non-contradiction (‘a circle is not a non-circle, such as a square’).⁹ *Illogical* is not the same as *a-logical* (non-logical). The former violates logical principles whereas the latter refers to all those aspects that are different from the logical-analytical aspect, that is, to the *non-logical aspects* of reality.

There are two decisive hallmarks of a concept:

- (i) The combination of a multiplicity of features into the logical unity of a concept;
- (ii) The fact that each one of these traits is universal.

Consider the concept of a *triangle* while keeping in mind the first hallmark of a concept. It is normally subsumed under the concept of a *polygon* and is supposed to be constituted by three corners (vertices), three straight line segments as sides, such that

the interior of the triangle is that part of a plane enclosed by the triangle, and its outside (its exterior – see Weisstein, Eric W. 2005. ‘Triangle.’ From *MathWorld*, A Wolfram Web Resource. Available at: <http://mathworld.wolfram.com/Triangle.html>).

The starting-point of this definition is the concept of a polygon, that literally combines numerical and spatial properties, for the concept *polygon* unites the terms *many* (‘poly’) and *corners* (implying *sides*). In the case of a *triangle* the number ‘3’ is used to restrict (i.e. to limit) the number of sides (i.e. straight line segments) of a polygon. The surface enclosed by a triangle depends on the spatial term ‘corners’ (the plural reflects the numerical meaning of multiplicity), in combination with the connecting sides (straight line stretches). The concept of a triangle thus unites a multiplicity of numerical and spatial properties captured by the terms ‘three,’ ‘straight line segments,’ ‘corners’ and ‘an enclosed surface’ (interior). Therefore it conforms to the first above-mentioned characteristic of a concept by combining a multiplicity of features into the unity of a logical concept.

Since every one of these properties that serve to constitute the nature of a triangle has a universal scope, i.e. applies *wherever* and *whenever*, the concept of a triangle at once also exemplifies the second hallmark of a concept. The conditions for being a triangle (i.e. the *law for* a triangle) hold *universally*. A concept is either directed at the (universal) *order for* reality or at the (universal) *orderliness (law-conformity) of* reality. It is possible to distinguish between concepts of *modal laws*, concepts of *type laws* and concepts of *things*. *Function concepts* account for the (modal) aspects of reality, while *type concepts* are focused on the many-sidedness of (natural and social) entities and processes.

An important consequence of hallmark (ii) is that whatever is *unique* and *individual* cannot be grasped in a concept. Concepts are *blind* to the unique and the individual.¹⁰ However, medieval philosophy did concede that our senses do give access to the individual. Later on, with the occurrence of the linguistic turn (late 19th early 20th centuries) exploring the individual is accomplished through the *deictic* function of language, its ability to *point at* something individual (*deixis*).

As noted, every attempt to define the characteristics united in a concept ultimately terminates in what is *indefinable*. The terms used in defining a triangle are derived from the numerical and spatial aspects and the core meaning of these aspects is indefinable and therefore *primitive*.¹¹ In this sense the acquisition of a concept is always based upon an insight that *transcends* conceptual knowledge,¹² at once highlighting the *self-insufficiency* of conceptual knowledge. We conceive ultimately by employing terms that we cannot conceive. Therefore conceptual knowledge is not properly understood unless it is related to *concept transcending knowledge*.¹³

Analytical skills between monistic isms and the importance of a non-reductionist ontology

It is certainly not sufficient to understand that concept-formation is constituted by the combination of the universal features of whatever is conceived, for the history of

philosophy and the disciplines portray an endless legacy of attempts to reduce the rich diversity within reality to some or other principle of explanation *in terms of which* everything in the universe could be explained. In Greek culture the Pythagoreans thought that they could explain everything in terms of the *ratio* between integers (i.e. in terms of rational numbers) – resulting in their slogan ‘everything is number’ (see Thesleff 1970, 82). The discovery of incommensurable ratios (i.e. irrational numbers) by Hippasus of Metapont (450 B.C. – see Von Fritz 1965, 288 ff.) caused a switch in the choice of a basic denominator, for a spatial figure like a rectangular triangle is perfectly limited and does not suffer from the *unlimited* nature of an irrational number such as the square root of 2. The importance of a non-reductionist ontology is best explained by briefly mentioning the diverse *ismic* orientations in some academic disciplines (natural sciences and the humanities). After this overview we shall focus on those philosophical distinctions that may enhance the analytical skills of scholars and help to prevent them from getting entangled in unsolvable theoretical *antinomies*.

The choice of alternative basic denominators within the various special sciences

The schools of thought mentioned below are by and large examples of trends that attempted to understand reality merely in terms of one (elevated) mode of explanation.

In *mathematics* we mention axiomatic formalism (Hilbert), logicism (Russell, Frege) and intuitionism (Brouwer, Heyting, Troelstra, Dummett).

Physics: Classical determinism (Einstein, Schrödinger, Bohm and the school of De Broglie) and the mechanistic main tendency of classical physics (last representative Heinrich Hertz) versus the Kopenhagen interpretation of quantum mechanics (Bohr and Heisenberg); the contemporary ideal to develop ‘a theory of everything’ (Hawking and super string theory: Greene).

Biology: The mechanistic orientation (Eisenstein), the physicalistic approach (Neodarwinism), Neovitalism (Driesch, Sinnott, Rainer-Schubert Solderm, Haas, Heitler), holism (Adolf Meyer-Abich), emergence evolutionism (Lloyd-Morgan, Woltereck, Bavinck, Polanyi) and pan-psychism (Teilhard de Chardin, Bernard Rensch); recent complexity theory (Behe’s notion of ‘irreducibly complex systems’ – see Behe 2003) and the idea of ‘intelligent design’ (see Dekker et al. 2006).

Psychology: The initial atomistic association psychology (Herbart), the stimulus-response approach, Gestalt-psychology [the Leipzig school (Krüger and Volkelt) and the Berlin school (Koffka and Köhler)], depth psychology (Freud, Adler, Jung), the logo-therapy of Frankl, phenomenological psychology, contemporary system theoretical approaches (under the influence of von Bertalanffy), and postmodern trends under the spell of the ‘linguistic turn’ (telling your ‘story’ may be healing).

The science of history: Compare the conflict between linear and cyclical conceptions of history, the Enlightenment ideal of linear accumulative growth, the

recurrence of the Greek conviction that history is eternally recurrent in the thought of Vico, Herder, Hegel, Goethe, Daniliwski, Nietzsche, Spengler and to a certain degree also Toynbee.

Linguistics: Two lines of thought dominated the 19th century – Rousseau, Herder, Romanticism, von Humboldt and the rationalistic trend running from Bopp, Schleicher, and ‘Jung-Grammatici’ to Paul (with his historicistic conception that views language as language-in-development). Cassirer, by contrast, developed his Neokantian theory of language (in which language is a thought-form imprinted upon reality), Bühler pursued the stimulus of behaviorism in his theory of signs, at the beginning of the 20th century Wundt dominated the scene, De Saussure contributed to the development of a structuralist understanding (followed by Geckeler, Coseriu and others), Reichling explored elements of Gestalt-psychology in his emphasis on the word as the core unit of language, Chomsky revived the doctrine of the a priori (that is what precedes experience) within the context of his transformative generative grammar.

Sociology: The initial organicistic orientation (Comte, Spencer) was continually opposed by mechanistic and physicalistic approaches (cf. L. F. Ward – late 19th century – and in the second half of the 20th century W. R. Catton), the dialectical heritage of Hegel permeated Georg Simmel’s formalistic sociology with its individualistic Neokantian focus (Park and Burgess explored this direction in the USA), Max Weber developed the sociological and economic implications of the Neokantian Baden school of thought, Talcott Parsons made the systems model (based upon von Bertalanffy’s generalization of the second main law of thermodynamics) fruitful for sociological thinking, opposed by conflict sociology (Dahrendorf, C. Wright Mills and Rex and by the Frankfurt school of Neomarxism), a systems theoretical approach was recently revived by J. C. Alexander, A. Giddens developed his structuration theory – and during the past two decades J. Habermas elaborated his theory of communicative actions.

Economics: The classical school of Adam Smith, the neoclassical approach (from Cournot and Dupuit to Menger, Jevons, Walras and Pareto), the marginalism of Marshall, Keynes’s ‘General Theory,’ as well as alternative approaches to competition (Chamberlin and Robinson).

The science of law: The historicistic orientation of the Historical School of von Savigny – followed by the Romanist (von Jhering) and Germanistic school (von Gierke), Neohegelianism (Binder), Neokantianism (Stammler, Radbruch, Kelsen), the revival of natural law theories after the second world war (Rommen, Stadtmüller, Küchenhof, Schmitt, Fuchs, Auer Coing, Maihofer and others), and legal positivism (which always seems to remain alive amongst legal scholars).

Theology: Dialectical theology (Barth, Gogarten, Brunner) in its dependence upon Kierkegaard and Jaspers, Bultmann (dependent on Heidegger), theology of hope (Moltmann – dependent upon the Neomarxism of Ernst Bloch), the historicistic design of Pannenberg (dependent upon Dilthey and Troeltsch), the ‘atheistic’ theology

of Altizer and Cox (influenced by Neopositivism), existentialist-hermeneutical trends (Fuchs, Ebeling, Steiger), theology of liberation (influenced by Neomarxism).

Reductionism is self-defeating because it results in antinomies

In order to side-step one-sided ismic orientations – both within philosophy and within the various academic disciplines – a well articulated, non-reductionist ontology is needed. The first hall-mark of such a view of reality is to accept both the *uniqueness* and the *coherence* between what is unique within reality. For example, throughout the history of mathematics it was either attempted to reduce space to number or number to space – but the third option was never explored: accept the uniqueness of the aspects of number and space and then investigate their interconnections (coherence). The currently still dominant arithmeticistic orientations of modern mathematics – claiming that spatial continuity has been reduced to the set of real numbers – is not self-critical about the antinomic nature of this claim. The understanding of real numbers employs the idea of the *actual infinite* (I prefer the phrase: *at once infinite*) which is dependent upon the irreducibility of the spatial order of simultaneity (at once), for it views any successive row of numbers *as if* it is given at once as an infinite totality.¹⁴ The arithmetization of the linear continuum argued for by Grünbaum¹⁵ is crucially dependent upon the employment of the actual infinite (see Grünbaum 1952, 300 and 302 where he claims that ‘the Cantorean line can be said to be already actually infinitely divided’ and where he concedes that his entire analysis is dependent upon non-denumerability¹⁶). Since the employment of the actual infinite presupposes the irreducibility of space it cannot afterwards be used to reduce space to number.¹⁷

Likewise, the concept of *matter* was delivered to mutually exclusive principles of explanation, varying from the Pythagorean claim that everything is number and the intermediate view that extension constitutes the ‘essence’ of matter [from the geometrization of mathematics in Greek culture up to Descartes (*res extensa*) and Kant (the proposition that bodies are extended is *analytic*) up to the mechanistic main tendency of modern physics since Newton and the eventual switch to an acknowledgement of the physical nature of material entities (stamped by the operation of energy in all its manifestations). What is lacking in this legacy is the insight that material entities have a function in the aspects of number, space, the kinematical and the physical without being exhausted by anyone of these aspects.

Another example is found in historicism (as it emerged at the beginning of the 19th century). Historicism claims that everything is embraced by the ever-changing flow of history – law, morality, art, even religion cannot escape from this changefulness of the (historical) world in which we live. Consequently, so the historicist argues, everything *is* history. However, the basic problem is that only that which in itself is not intrinsically historical and transient can *have* a history. If law is nothing more than a historical event that once ‘happened’ then we have left it behind and cannot any longer speak of *legal history*. It is only when we accept the constant structure of the jural aspect of reality that we meaningfully can speak of the significant changes that

took shape in the historical development of law (in South Africa we inherited Dutch-Roman law). The irony of all monistic *isms* is that they always achieve the opposite of what was aimed for. In the case of the historicist: if everything *is* history there is nothing left that can *have* a history and therefore history itself is also uprooted!

Scientific truth is more-than-logical

Immanuel Kant knew that the logical principle of contradiction at most can claim that two mutually exclusive statements cannot both be true at once (see Kant 1787 – B, 84). In order to establish the truth or falsity of premises or conclusions is therefore dependent upon extra-logical *grounds* or *reasons*, transcending the boundaries of logic. This caused Leibniz to introduce what he designated as the principle of sufficient reason (principiū ratiōnis sufficientis). Of course there is a difference between the *validity* of an inference and the truth of either its premises or conclusion. If we have the proposition ‘All living entities have 8 legs’ and another one stating ‘Human beings are alive,’ then the inference ‘Human beings have 8 legs’ is *valid*, in spite of the fact that one premise and the conclusion are *false*.

Antinomy and contradiction¹⁸

Antinomies actually only come to light within the context of a more-than-logical view of reality because they result from confusing different aspects of reality. It concerns a conflict between the laws of *different* spheres of our world – anti = against and nomos = law. Literally it concerns a clash of laws (*law-spheres* or *modal functions*).¹⁹ The *inter-modal* nature an antinomy, reflecting a denial of what are unique and irreducible aspects of the world, always implies a logical contradiction, but the latter does not necessarily presuppose an antinomy. Confusing two spatial figures – such as a square and a circle in the illogical concept of a square circle – concerns two figures occurring *within* one aspect, the aspect of space. The historicist claim, by contrast, confuses different aspects of reality and is therefore a victim of an inter-modal antinomy. Yet this antinomy still entails a logical contradiction: everything is history if and only if everything is not history.

The mistaken opposition of analysis and synthesis

We noted that the discovery of irrational numbers caused Greek mathematics to switch to a geometrical perspective and that this shift resulted in the long-standing view that the ‘essence’ of matter is found in its *extension*. Descartes holds that ‘the nature of body consists not in weight, hardness, colour, and the like, but in extension alone’ (1965, 200). Kant continued this view in his conviction that the judgment ‘All bodies are extended’ is analytic, whereas the judgment ‘All bodies are heavy’ is synthetic (Kant 1787 – B, 11). In the course of the 19th century logicians started to realize that the characteristic mass (*‘weight’*) is also *analytically* implied in the concept of a physical body. Consequently also the judgment ‘All bodies are heavy’ equally has an analytical side for if the predicate mass is not contained in the concept of a body

predicating it afterwards would be illogical. If a proper concept of a physical body does not imply (some or other specified form) of this physical feature (mass/weight) in an analytical way to begin with, it cannot afterwards be predicated of the body, except illogically. [From: P is non-Q, one cannot infer: P is (such and such) Q.]

Moreover, from the above-mentioned sketch explaining the synonymy of *analysis* and *abstraction* it is clear that *identification* is just one of the two ‘legs’ of analysis (the other being *distinguishing*). But since identification is nothing but the ‘bringing together’ (synthesizing) of a multiplicity of traits, synthesis, just being one ‘leg’ of analysis, is not opposed to analysis – its correlate is distinguishing. Therefore it is mistaken to oppose *analysis* and *synthesis*. From Figure 2 another important facet of analysis and abstraction is seen, namely that identification and distinguishing always occurs on the basis of discerning similarities and differences. One may even speak of the *circle of comparison* in this context. Analogous to the *hermeneutical circle*²⁰ the circle of comparison implies that identification and distinguishing can only take place on the basis of similarities and differences, while discerning similarities and differences in turn presuppose an analytical *act* of identification and distinguishing.

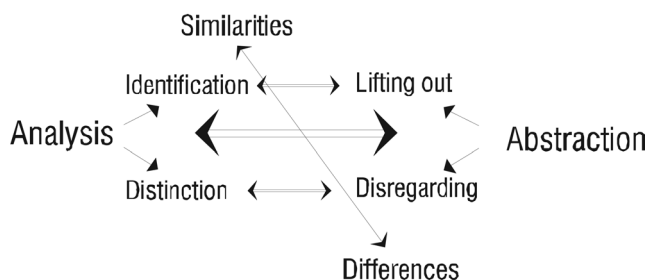


Figure 2:

Once more: critical skills

The presupposition for exercising critical skills is therefore given in an explicit awareness of the *criteria* guiding any critical endeavours. The first step of a critical encounter should comprise *immanent criticism* such as launched above against arithmeticism and historicism. In both cases the immanent criticism unveiled the intrinsic antinomic nature of the positions assumed by these two *reductionistic* approaches. Therefore the ontological principle of the excluded antinomy (*principium exclusae antinomiae*) ought to guide the critical skills of scholars, keeping in mind that this principle is foundational to the logical principle of non-contradiction. Training both informal and formal logic may further enhance the critical skills of scholars, helping them to develop a sound argumentative style resulting in a consistent and contradiction-free way of writing.

In addition to these considerations we merely note three further important criteria for critical thinking and their enhancement. First of all there is the requirement that appreciating what is worth-while in the thought of a scholar should precede the ideal

of being critical – it amounts to a sense of *critical solidarity*. In the second place *factual critique* is important in terms of the requirement that scholarly work ought to be *up to date* with the *states of affairs* within a particular discipline. And in the third place – after a sense of solidarity has been shown, immanent criticism formulated and factual criticism exercised – one can proceed by investigating the theoretical framework (paradigm) and ultimate commitments of a thinker. Yet, these additional perspectives require a broader treatment in another article and therefore will not be further explained here.

Epistemic values guiding critical analytical skills

Because we have ‘positioned’ *analysis* within the context of the various (ontic) aspects of reality – compare the designation *logical-analytical aspect* – it opens the way to explain its meaning with reference to its coherence with other aspects – in particular the aspects of number and space. If we designate the actual function we have as human beings within the logical-analytical aspect as *epistemic* it is possible to discern many more guidelines for critical thinking. What immediately comes to mind is the *epistemic values* that surfaced in 20th century discussions in the philosophy of science. Since the rise of the Baden school of Neokantian thought at the beginning of the 20th century (Windelband and Rickert), the philosophical legacy of the West has become accustomed to speak of *values* (often as a substitute for principles or norms). Kuhn distinguishes between the *application* of rules and the act of *evaluating* (see Kuhn 1977, 331 and 1984, 379). He mentions five values influencing theory choice: ‘accuracy, consistency, scope, simplicity, and fruitfulness’ (Kuhn 1984, 373). McMullin follows Kuhn’s view of epistemic values and also discusses the choice of a theory in terms of ‘value-judgements’ which differ from the mere application of a rule (cf. McMullin 1983, 11). He preferably speaks of ‘epistemic values’ and transforms the values mentioned by Kuhn by referring to them as predictive accuracy, internal coherence, external consistency, unifying power and epistemic fertility. Epistemic simplicity is added to this list (McMullin 1983, 15–16).

It is clear that this way of dealing with ‘epistemic values’ is dependent on underlying coherences between the analytical facet of theory formation and diverse non-analytical aspects of our experience. For example, ‘fertility’ first reminds us of a biotic phenomenon. Plants need ‘fertile soil’ in order to *grow* properly and *bear fruit*. Analogously, theories may turn out to be ‘fruitful’ by bearing ‘fruit’. In the value of *epistemic fertility*, we therefore meet a biotical analogy within the structure of theoretical thought, i.e. within the structure of (deepened) analysis (recall our remark regarding a numerical unity and multiplicity and a logical-analytical unity and multiplicity – an instance of a numerical analogy within the structure of the logical-analytical aspect).

It is clear that these ‘epistemic values’ depend upon the coherence of the analytical mode with the other (unique) aspects of reality. Quine indirectly explores the universal scope of the logical-analytical aspect with reference to the disciplines of

logic and *mathematics*. He claims that ‘the relevance to all science and their partiality toward none’ are ‘two traits of logic and mathematics,’ which makes it possible ‘to draw an emphatic boundary separating them from the natural sciences’ (Quine 1970, 98). Already Frege has employed the term ‘logical’ in order to designate a kind of generality or universality that is not dependent on the specific nature of any kind of entity (see Dummett 1995, 224). But from our remark about epistemic fertility it must already be clear that this aspectual universality does not entail that the *meaning* of the logical-analytical aspect could be separated from its coherence with the other (non-logical, ontic) aspects of reality. The mere fact that Hilbert remarked that it is necessary to come to ‘a partially simultaneous development of the laws of logic and arithmetic’ (Hilbert 1970, 199), shows that the meaning of analysis has its ontic foundation in the arithmetical aspect – even though an analysis of the meaning of number requires an actual human function within the logical-analytical mode. And this mode intimately coheres with the quantitative meaning of the numerical aspect. Quine correctly affirms: ‘Any logic has to come to terms somehow with quantification, if it is not going to stop short’ (Quine 1970, 88).

The idea of epistemic values therefore merely underscores the general perspective argued in this article, namely that the meaning of analysis only comes to expression in and through the inter-modal coherence between the logical-analytical aspect and the other aspects of reality.

NOTES

- 1 This article should be seen as an explanation of the necessary background perspectives of the Paper that was read at the HELTASA Conference on November 21, 2007 at the *Central University of Technology* in Bloemfontein (on: *Scholarly Communication*).
- 2 See *Gasette*, October 5, 2007 as well as the *Higher Education Act*, 1997 (Act No. 101 of 1997).
- 3 In a different context he says: ‘The idea of the continuum is a geometrical idea, expressed by analysis in the language of arithmetic’ (Bernays 1976, 74). We shall return to this remark below.
- 4 It should be kept in mind that *atomism* is one of the most pervasive *analytical strategies* literally found in the history of all the disciplines (natural sciences and humanities)
- 5 Cantor views a *set* as any combination M into a whole of particular, properly distinct objects m (which are called the *elements* of M) of our intuition or of our thought. In German: ‘Unter eine “Menge” verstehen wir jede Zusammenfassung M von bestimmten wohlunterschiedenen Objekten m unsrer Anschauung oder unseres Denkens (welche die “Elemente” von M genannt werden) zu einem Ganzen’ (Cantor 1962, 282).
- 6 We are all familiar with *classification* – just think of a normal drawer with folders and documents (also reflected by the operating system of contemporary computers. Yet classification is simply an instance of the meaning of analysis, that is, of identification and distinguishing.
- 7 These principles stipulate that in what is analyzable A is A and A is not non- A .
- 8 Russell defines number with the aid of his supposedly purely logical *class* concept. The logical concept, he claims, enables the reduction of mathematics to logic. For example,

the number '2' is 'defined' in the following way: '1 + 1 is the number of a class w which is the logical sum of two classes u and v which have no common terms and have each only one term. The chief point to be observed is, that logical addition of numbers is the fundamental notion, while arithmetical addition of numbers is wholly subsequent' (Russell 1956, 119). The irony, however, is that Russell used the meaning of number in order to distinguish between different ('logical') classes, for how else can he speak about the sum of 'two' classes where each of them contains 'one' element. What is presupposed is an insight into the quantitative meaning of the numbers '1' and '2'! Consequently, the number '2,' which had to appear as the result of 'logical addition,' is presupposed by it!

- 9 Keep in mind that figurative language use (i.e. the metaphorical employment of words) can apparently side-step the restriction of pure logic, for we are all used to refer to the following 'square circle' – a *boxing ring*!
- 10 For that reason it is oftentimes attempted to trace a criminal with the aid of a so-called 'identity-kit'. That is to say by presenting a visual image to people in order to find a unique individual. Sensory images capture the unique individuality of a person – positing concepts simply introduces universal categories (such as 'a human being,' 'a man,' and so on.
- 11 The core meaning of an aspect at once also brings its *irreducibility* to expression. It is reflected in its indefinability, explaining why it is designated as *primitive*. Korzybski also underscores that one cannot define *ad infinitum*: 'We thus see that all linguistic schemes, if analysed far enough, would depend on a set of "undefined terms". If we enquire about the "meaning" of a word, we find that it depends on the "meaning" of other words used in defining it, and that the eventual new relations posited between them ultimately depend on the ... meanings of the undefined terms, which, at a given period, cannot be elucidated any further' (Korzybski 1948, 21).
- 12 Paul Bernays, in a relatively concise contribution to the Volume on *The Philosophy of Karl Popper* (published in the Series, *The Library of Living Philosophers*), argues that the constitutive part of *rationality* is given in the *conceptual* element. He states that the 'proper characteristic of rationality' is 'to be found in the conceptual element' (Bernays 1974, 601).
- 13 Explaining what concept-transcending knowledge involves will not be done in this article. An analysis of conceptual knowledge and concept-transcending knowledge is found in Strauss 2003.
- 14 We argued that the whole-parts relation derives from the core meaning of the spatial aspect.
- 15 Grünbaum has combined insights from the theory of point-sets (founded by Cantor) with general topological notions and with basic elements in modern dimension theory in order to arrive at an apparently consistent conception of the extended linear continuum as an aggregate of unextended elements (1952, 288 ff.).
- 16 Cantor's proof of the non-denumerability of the real numbers collapses without the assumption of the at once infinite and the at once finite, as we stated, only has meaning if space is irreducible to number.
- 17 For a more detailed analysis see Strauss (2005, 66–72).
- 18 An extensive analysis of this distinction is found in Strauss 2007.
- 19 Aspects are also designated as *modalities* because they relate to the *how* of the universe and not to the concrete what – and from Latin we know that the *how* is designated by the

term *modus* (mode). The aspects of reality represent therefore multiple *modes of being*, frameworks within which concrete (natural and social) entities function.

- 20 Interestingly, the so-called hermeneutical circle (analogically) simply explores the two sides of the spatial whole-parts relation: on the one hand it is stated that one can only understand the meaning of a *part* in terms of an understanding of the *whole* while on the other it is claimed with equal force that the *whole* can only be understood in terms of understanding its (constitutive) *parts*!

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