INFINITY: BETWEEN DIVINITY, TIME, AND ETERNITY

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ABSTRACT:

In following in the footsteps of the idea of infinity it will be shown that assigning a divine status to it accompanied the idea of infinity in early Greek philosophy. Yet, subsequently it was followed up by a dialectical process slowly moving in the direction of a de-divinization of infinity. This process initially shied away from describing God as infinite (in the sense of being unlimited), for the question was how would God be able to engage in self-contemplation if God's essence is infinite? However, Plotinus, Gregory of Nyssa and Augustine started to differentiate between a succession of numbers (accessible to human beings) and observing such an infinite multiplicity all at once, without any before and after (a capacity ascribed to God only). Much later Maimon continues this legacy, for he assigns the *successive infinite* to a finite mind and the *at once infinite* to an absolute mind. Wittgenstein captures the heritage of eternity as the timeless present in his own way when he remarks that if eternity does not mean infinite temporal duration but timelessness, then eternal life belongs to those who live in the present. The distinction between the *successive infinite* and the *at once infinite* in a certain sense also characterizes the difference between the two mathematicians Cantor and Weyl. The former does acknowledge the mathematical status of both the *successive infinite* and *at once infinite*, but then introduces the absolute infinite which belongs to God. Since he rejects the mathematical employment of the *at once infinite*, Weyl reserves the actual infinite for God. Yet, Weyl is certainly correct in characterizing mathematics as the science of the infinite. While the axiomatization of mathematics succeeded in avoiding the antinomies Gödel has shown (in 1931) that consistency entails incompleteness. Consequently, Hilbert had to acknowledge that the insight of consistency is rather to be attained by intuitive reasoning which is based on evidence and not on axioms. Infinity will always occupy a central position in mathematics and it will constantly prompt mathematicians to account for the difference between the *successive infinite* and the *at once infinite* in terms of the interconnections between number and space.

Keywords: idea of infinity; eternity, timeless present; antinomies; incompleteness; intuitive reasoning; successive infinite; at once infinite; number and space

1.   THE URGE TOWARDS THE INFINITE

Although the ability to *count* early surfaces in the development of children, humans are not immediately inclined to explore the succession of numbers. I remember that a playful challenge regarding counting prompted me the year before I entered school to overtake the older children present. We were invited to see who can count the quickest and the furthest. But by the time I realized that this is a serious counting contest, some of the others already counted beyond thirty. I paused for a while, assessing my options and then decided to surpass all the contenders immediately. So I started to count: "one-uncountable" (*ontelbaar*); two-uncountable, three-uncountable; and so on. Of course, I was disqualified because the others objected that "ontelbaar" is not a *number*. [1]

Little did they (and I) know that "telbaarheid" [*countability*] became a key feature in the comparison of (infinite) sets. Galileo already played with correlating certain (partially overlapping) rows of numbers in 1638. He compared the number of squares (1, 4, 9, 16, …) with the natural numbers (1, 2, 3, 4, …) and realized that the further one pursues this path the lesser the squares appear to be (10 between 1 and 100 – 1/10th; 100 between 1 and 10000 – $1/100^{th}$; 1000 between 1 and 1000 000 – $1/1000^{th}$, and so on). However, by reversing the procedure one may ask how many roots of squares are there? The number 1 is the root of 1^2; 2 is the root of 2^2; 3 is the root of 3^2; and so on. This means that there are just as many squares as natural (counting) numbers, for every natural number is the root of a square.

2.   INFINITY AND THE WHOLE-PARTS RELATION

Galileo therefore warned against arithmetical operations when infinity is involved, for apparently one cannot meaningfully speak about *smaller as*, *larger than* or *equal to* in the case of the infinite. However, it was Bolzano who explored this issue further in his posthumously published work on paradoxes of the infinite (*Paradoxien des Unendlichen* – 1851). In §20 Bolzano sets out to discuss a remarkable relationship between two infinite sets (*Mengen*), namely that it is possible to relate in a one-to-one way a whole set and a proper subset to each other:

> I assert namely: two sets that are both infinite could stand in such a relationship to each other, that on the one hand it is possible to correlate everything belonging to the one set with everything belonging to the other into a pair such that no single thing appears in two or more pairs; and in addition is it on the other hand possible that one of these sets contains the other one as a mere part, so that the multiplicities

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[1] Interestingly every kind of bird has a specific number of eggs in its nest. Plovers, for example, have 4 eggs and doves 2. When one of these eggs is removed the bird will lay another one but when nothing is removed the ovary will stop generating additional eggs. This is a non-human instinctive form of counting! ("Jede Vogelart hat eine oft recht genau bestimmte Zahl von Eiern in einem Nest, z. B. stets *4* bei Kiebitzen, 2 bei Tauben. Nehme ich immer wieder ein Ei weg, so legt der Vogel weiter, lasse ich die volle Zahl - so stoppt der Eierstock seine sicher unbewußte Eiererzeugung" – Portmann 1973:21).

which they represent when we observe them as the same, that is to say as units having the most diverse relationships to each other (Bolzano 1851:28). [2]

This demonstrates that the time-honoured slogan, namely that the whole is more than (or greater as) the sum of its parts, does not hold for infinite sets, because in the case of the infinite the whole could be *equivalent* to a part. [3] However, this is only the case when (infinite) sets are at stake, for in his *Foundations of Geometry* (1899) Hilbert upholds the axiom (3:8): "Das Ganze ist größer als der Teil" (*That the whole is greater than a part*) (see Volkert 2015:3).

The implication is that infinity reveals a connection to the whole-parts relation. Aristotle phrased this stipulation regarding the priority of the whole in terms of his view of the Greek city-state (*polis*) as follows: "Therefore, the state, by virtue of its nature, precedes the family and the individual, for the whole must precede the part." [4]

3.   IS THE UNIVERSE FINITE OR UNLIMITED?

The other side of this coin is found in the question whether or not the universe is finite or unlimited. We know that light travels at 300 000 km per second (which is about seven times around the earth in one second). Moore uses this fact to give us an appreciation of the vast spatial expansion of the physical universe (incidentally in the same year in which the *Hubble Space Telescope* was launched). He remarks:

> On a clear moonless night a faint patch of light can be seen in the constellation Andromeda. This is the Andromeda nebula. It is a galaxy of about hundred thousand million stars, each of them a sun like our own. Its light takes some two million years to reach us. It is the farthest object visible to the naked eye. Yet by comparison with other galaxies it is a close neighbour (Moore 1990:xi).

Although Greek philosophy was hesitant to accept the idea of God's infinity, medieval and modern philosophy explored it in their contemplation, as will be explained presently. In passing is should be noted that the idea of the infinity of God still features in the context of time, immortality and eternity. In a recent work on "*Theologie und Naturwissenschaften*" we read:

> Where the appearance of a transcendence or relation to God is understood as an indication of the creation of the soul, the soul is not pre-existent and a part or essence of God; for a relation commences, and this relation, named soul, is only immortal insofar as it is related to the immortal God. The human soul connects the

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[2]   "Ich behaupte nämlich: zwei Mengen, die beide unendlich sind, können in einem solchen Verhältnisse zueinander stehen, daß es einerseits möglich ist, jedes der einen Menge gehörige Ding mit einem der anderen zu einem Paare zu verbinden mit dem Erfolge, daß kein einziges Ding in beiden Mengen ohne Verbindung zu einem Paare bleibt, und auch kein einziges in zwei oder mehreren Paaren vorkommt; und dabei ist es doch andererseits möglich, daß die eine dieser Mengen die andere als einen bloßen Teil in sich faßt, so daß die Vielheiten, welche sie vorstellen, wenn wir die Dinge derselben alle als gleich, d. h. als Einheiten betrachten, die mannigfaltigsten Verhältnisse zueinander haben" (Bolzano 1851:28, see page IX of the Contents).

[3]   The idea of a whole or totality acquires a particular significance when it is related to the idea of *infinite totalities*.

[4]   κ ὶ   ἐ ῇ ύ ό ῇ ὶ ί   ὶ ἕ   ἡμῶ ἐ ί . ὸ ὰ ὅ   ἀ   ῖ ῖ
t ῦ μὲ   – Aristoteles, *Politica*, 1253 a 19-20 (Aristoteles 1894:149).

finiteness of the human being, insofar as it has a beginning, with the infinity of God, insofar as it is immortal. [5]

4.   CREATED ETERNITY?

This position is reminiscent of the medieval idea of a *created eternity*. Dooyeweerd notes that according to the medieval view "[t]ime can only be experienced in its relation to *created eternity* (the *aevum*, as it is called in Scholasticism, in opposition to the *aeternitas increata*, the uncreated eternity of God)" (Dooyeweerd 2017:5). Distinguishing between time and eternity then boils down to time that has a beginning and an end whereas eternity lacks both. The *aevum* does have a beginning but no end (Dooyeweerd 2017:20-21). Dooyeweerd states that he "would like to take over the term '*aevum*' in the sense of an intermediate state between time and eternity" and then specifies the way in which he understands it:

> As an *actual condition* the *aevum* therefore is nothing but the creaturely concentration of the temporal upon eternity in the religious transcendence of the boundary of time (Dooyeweerd 2017:22).

From the preceding brief explanation it must be clear that the idea of infinity permeates diverse fields of knowledge. Its importance for human intellectual endeavours is revealed in the words of the foremost mathematician of the early twentieth century, David Hilbert:

> From time immemorial, the infinite stirred human emotions more than any other question. Hardly any other *idea* has stimulated the mind so fruitfully. Yet no other *concept* needs clarification more than it does (Hilbert 1925:163).

This assessment not only applies to mathematics and physics (astronomy), but also to the role which the notion of infinity played within the discipline of theology, whether in a negative sense or a positive sense. In order to understand the historical roots of the dominant conceptions of infinity we therefore have to commence with what happened with infinity in Greek culture.

5.   INFINITY IN GREEK PHILOSOPHY

Although most of the prominent nature philosophers in Greece opted for a distinct principle of origin, such as water, fire or air, Anaximander advanced a view in which this principle of origin was designated as the *apeiron* – the *infinite/unlimited*. The genesis and origin of existing things is the *apeiron*, translated by Diels-Kranz as "das grenzenlos-Unbestimmbare" (what is limitless-indeterminate – B Fragment 1). The second B Fr. asserts that the *apeiron* "is without ageing" while the third B Fr. adds that the *apeiron* is without death and destruction.

Zeno was the first philosopher exploring the whole-parts relation (see B Fr. 1-3 and Hasse et. al., 1928:9-10). This entails a "turning inwards" compared to the most basic understanding of

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[5]   "Wo das In-Erscheinung-Treten eines Transzendenz- oder Gottesbezugs als ein Indiz für die Erschaffung der Seele verstanden wird, ist Seele nicht präexistent und Teil oder Wesen Gottes; denn eine Beziehung beginnt, und diese Beziehung, Seele genannt, ist nur insofern unsterblich, als sie Beziehung zum unsterblichen Gott ist. Die menschliche Seele verbindet die Endlichkeit des Menschen, insofern sie einen Anfang hat, mit der Unendlichkeit Gottes, insofern sie unsterblich ist" (Lüke, 2014:227). Much later Hegel contemplated the *infinity* of what is *finite* (see Griffioen 1976:70 ff.).

infinity in the sense of an *endless succession*. It appeared now that one of the crucial features of continuity is given in its infinite divisibility. Anaxagoras explored these perspectives further in his B Fragments 3, 6 and 8 (see Scholz 1928:42 ff.).

6.   ETERNAL FLUX OR STATIC ONTIC FORMS?

However, it should be kept in mind that Greek philosophy from its inception wrestled with its dualistic basic motive, the motive of *matter* and *form*, as it was designated by Aristotle. Initially the matter motive attained the upper hand, evinced in the emphasis on the eternal flowing stream of life which passes through all visible forms. In the thought of Empedocles the one, undivided form of being assumed by the school of Parmenides, was split up into four static ontic forms, but their coming together and splitting apart were still governed by the two dynamic flowing forces of the *philia* (*love* – good) and the *neikos* (*strife* – bad). This shows that the matter motive was only partially de-divinized. Anaxagoras gave the decisive step towards the *Nous* (reason) as an autonomous form principle which is not mixed with anything (Diels-Kranz B Fr. 12).

For Anaximander the divine was predicated as being infinite. With the shift of primacy from the matter motive to the form motive the intimate link between matter and infinity was continued even though matter was de-divinized. Both Plato and Aristotle are in the grip of the ultimate split between matter and form as mutually exclusive ultimate principles of origin. Happ is therefore fully justified in his assessment that between matter and form there is an irreducible original opposition or dualism present: The "highest matter" cannot be reduced to the "highest form": "As in Plato and the Academy an original opposition [*Ur-gegensatz*] here continues to exist." [6]

7.   NEGATING INFINITY AS A PREDICATE OF GOD

Mühlenberg characterizes this legacy in related terms when he claims that the philosophy of Plato and Aristotle unfolds on the basis of negating infinity as a predicate of God. According to him infinity was exclusively assigned to matter (Mühlenberg 1966:28). Yet later on, in connection with the view of Plotinus, Mühlenberg remarks that whatever is determined presupposes undetermined matter as its stuff. [7]

Within the negative theology of the middle ages, exploring the argumentation of Plato's dialogue *Parmenides*, the dialectical view entailed in it proceeds from the assumption that the Eleatic hypothesis that the one truly *is* – in the sense of having metaphysical *being*. As this hypothesis is developed in *Parmenides*, Plato argues that "if there is *one*, there must also be

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[6]   "Darüber hinaus geht etwa Owens, wenn er van einer Pros-hen Einheit der vier aristotelischen Ursachen' spricht, also de facto alles auf die 'Form' reduzieren will. Man kann das Form-Sein 'pros-hen' auf die höchste Form beziehen und die Weisen des Materie-Seins auf die oberste Materie, aber nicht die oberste Materie auf die oberste Form: Hier bleibt – wie bei Platon und in der Akademie – ein Ur-Gegensatz bestehen (Happ 1971:805, note 628). "Owens exceeds this level when he speaks about a *Pros-hen* unity of the four Aristotelian causes, therefore de facto aiming at reducing everything to 'Form'. One can relate the Form-Being ('pros-hen') [focal meaning] to the highest form and the modes of Matter-Being to the highest matter, but not the highest matter to the highest form: Here remains – as with Plato and within the Academy – an original dualism in effect."

[7]   "Denn alles Bestimmen setzt die unbestimmte Materie als seinen Stoff voraus" (Mühlemnerg 1966:129).

number" and "if there is number, reality will be many, indefinitely numerous; number emerges as unlimited and having existence" (Plato 1961:26-27; *Parmenides* 144). Dooyeweerd explains that "the *one* is first of all taken in the abstract, absolute sense that had been given to it by the Eleatics (and, in the first place, by Parmenides), that is, as a unity that excludes all plurality, motion, change, becoming" (Dooyeweerd 2012:200). This explains why for Plato the *one* as absolute unity is pure *peras* (delimitation). Yet the *apeiron* (unlimited) does not originate in the *monas*, for ultimately the thought of Plato proceeds from the basic spit between the limited and the unlimited / infinite. Happ alludes to the two domains in Plato's thought as follows:

> Adjacent to his earlier distinguished two spheres of being, the noetic domain of eternal immutable (ideas) and the sensory perceivable province of   óμ [becoming, being generated], Plato now introduces another hardly comprehensible third domain. [8]

8.   INFINITY IN THE THOUGHT OF ARISTOTLE

Aristotle understood two basic features of infinity as they are related to number (succession / addition) and space (divisibility). He remarks: "Further, everything that is infinite may be so in respect of addition or division or both." On the same page he explains that "the infinite cannot be an actual thing" (Aristotle 2001:260; *Phys.* 204a4-5; 204a20 ff.). The same position is assumed in his *Metaphysics*: "And evidently the infinite cannot exist actually. For then any part of it that might be taken would be infinite (Aristotle 2001:866; *Metaph.* 1066a11 ff.). Moreover, the infinite is immediately related to continuity and infinite divisibility in Book III of his *Physica*: "and the infinite presents itself first in the continuous—that is how it comes about that 'infinite' is often used in definitions of the continuous ('what is infinitely divisible is continuous') (Aristotle 2001:235; *Phys.*200b13-19). Later on, it is asserted that something continuous must be infinitely divisible: "it must be divisible either into indivisibles or into divisibles that are infinitely divisible, in which case it is continuous" (Aristotle 2001:316-317; *Phys.* 231b12-15).

Mühlenberg holds that Aristotle exclusively locates infinity in *matter* ("das Unendliche ausschlie lich der Materie zuweist" – Mühlenberg 1966:28). To this he adds the remark that it cannot be proven that the statement asserting God's infinity is an exclusively Christian idea.

9.   ON THE WAY TOWARDS ASSIGNING INFINITY
    TO GOD

For this reason, Mühlenberg discerns in the thought of Gregory of Nyssa (335-394) for the first time in the history of philosophical and Christian thinking that the predicate *infinite* is applied to God. Of course, this development had to face the classical idea that God is *limited* – one of the most prominent features of the theology of Origen (184-253). Mühlenberg mentions Origen's *De Princ.* II,9,1 (Mühlenberg 1966:26). The self-contemplating God of Aristotle would be unable to comprehend itself if it was not limited, i.e. if it was infinite. Later on, Georg Cantor, the founder of transfinite arithmetic, called upon Origen to support his own view of the

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[8]   "Platon führt neben den schon früher unterschiedenen zwei Seinsbereichen, dem noetischen des ewig Unwandelbaren (Ideen) und dem sinnlich wahrnehmbaren der   óμ [become, be born], nun einen dritten, schwierig zu erfassenden Bereich ein" (Happ 1971:98).

actual infinity of God: "Origines always only had the ἄ   [apeiron] in view and states that when the divine power was ἄ   then God would not have been able to know itself." [9]

Plotinus (204-270), Gregory of Nyssa (335 – 395) and St. Augustine (354-430) gave direction to the subsequent reflections on these issues. Plotinus explored the negative inclination found in Plato's dialogue *Parmenides*.

In passing we may note that, similar to the Greek conception of concrete spatial extension, according to which a body is its place, one finds a similar "materialized" understanding of time in the old oriental cultures:

> The lawlike observation of star constellations led during the past thousand years to the development of a (mathematically founded) prognostic astronomy. Although, as far as one can see, not conceptualized in the abstract, 'time' in the old oriental cultures is conceived as constituents of the cosmos, as a materialized structure. [10]

10.   PLOTINUS: A DIALECTICAL VIEW ON THE INFINITE

In his *Enneads* Plotinus follows in the footsteps of Plato. The first antinomy discussed by Plato (and Plotinus) in the *Parmenides* of Plato assumes that the One is without any multiplicity, that it is *absolutely one*. But then nothing positive can be said about it, such that it is a whole or that it moves. A whole is that in which no part is lacking – but if all the parts are present wholeness entails multiplicity, thus contradicting the supposed unity of the One. Thought through consistently nothing positive can be said of the absolute One.

Likewise, in the fourth antinomy the Other becomes a victim of the same fate, for Plato argues that the Other does not have unity, duality or multiplicity (i.e. number) – matter is not embodied (compare the view of Plotinus in his *Enneads* II,4,9).

So, as soon as the dialectical understanding of the origin is thought through in a positive fashion, it entangles thinking in the antinomic affirmation and denial of all properties both in respect of the One and the Many – or thinking terminates in the total negation of all determinations of being (or conceptual determinations).

What is said about the One and about matter is practically the same. If indeed all other particulars are abstracted, then the reader without any reservation might have concluded to the complete identity of the One and the many. But that would have twisted the true intention of Plotinus awkwardly.

The One is negatively characterized as *motionless* (*Enneads* V,2,1) and as without *number* (*En.* V,5,4; V,5,11). Furthermore, the One is *without limit* and *size* (*En.* VI, 7, 32) and *without form* and *shape* (*En.* V,5,6; V,5,11; VI,7,32; VI,7,33, VI,9,3).

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[9]   "Origines hat immer nur das ἄ   [apeiron] im Augen und sagt, wenn die götliche Kraft ἄ   wäre, könnte Gott sichselbst nicht erkennen" (Cantor 1962:403).

[10]   "Die regelhafte Beobachtung von Gestirnkonstellationen führte im Laufe der vortgeshcrittenen 1. Jahrtausends in die Entwicklung einer (mathematisch fundierten) prognostischen Astronomie. Wiewohl, soweit man sieht, nicht abstract konzeptualisiert, wird 'Zeit' in den alorientalischen Kulturen als Konstituente des Kosmos, als materialisierte Struktur erfasst" (Hegenbarth-Reichhardt, In: Kratz & Spieckermann, 2009:41).

When these designations are compared with the description of matter (in relation to the fourth hypothesis of Plato's *Parmenides*), then it is striking that almost no difference is found. What is said about the One and matter is practically the same, thus misleading the reader to think that the One and the many are the same. However, in order to highlight the true *dialectical opposition* of the One and matter Plotinus had to revert to *positive* stipulations.

What is important for our analysis is the fact that Plotinus developed a re-appraisal of the actual infinite. It is captured in *En*. III,7 in connection with his understanding of eternity as the timeless present (see the entire work of Beierwaltes, 1967). Given the impact of the dialectical basic motive of matter and form the Greek legacy is not at ease with the identification of *perfection* and *infinity*. Therefore, Diekamp is insensitive in respect of this trait of Greek philosophy when he asserts that the concept of pure being results in the highest perfection and infinity (Diekamp 1962:204).

A point of connection for attributing infinity to God rather pursued the alternative path of the *immutability* of what is *unchangeable*. The static, supra-sensory ontic forms in Plato's theory of ideas is motivated by the urge towards what is indestructible – as opposed to the sensory world of becoming (*genesis*). In the thought of Plotinus the dialectical effect of the ground-motive of matter and form also manifests itself in the opposition of the *One* and *matter*. Spatial designations are used to circumscribe the ultimate opposition between the *One* and *matter*. The One is always portrayed as being elevated above all being (form, measure and delimitation), while matter, by contrast, is seen as a permanent substrate (*hupokeimenon*, *En*. II,4,4,8; cf. II, 4,6,4 and 19). In this context we also find the opposition of light and darkness respectively identified with the One and *matter* (*En*. VI,8,18). The dualistic understanding of the origin in Plotinus's thought is also depicted by means of the act-potency scheme, in the sense that the One contains everything proceeding from it. Potentially it contains everything. The potentiality of the One, however, is immediately delineated from the *receptivity* of matter. Identifying the two will incapacitate the source – the negation of production (*En*. V, 3,15). Consequently, this productive activity of the One is designated as the primary (first) activity (*En*. V,6,6,6-8).

Things are constituted by matter and (form-)idea (*En*. I1,4,6). The two ultimate poles in the hierarchy of being continues to reveal the theoretical elaboration of the supra-theoretical basic motive of form and matter. The split between the One and Matter, Good and Evil, Beauty and what is Ugly, Light and Darkness, Beginning and End, High (elevated) and Low (underneath) even finds expression in the opposition of the *One* and the *Many* (cf. *En*. III,2,2). Through this the second *hypostasis* is threatened, for on the one hand Plotinus posits that evil does not belong to the sphere of being (*En*. 1,8,3) and on the other hand the *Nous* entails multiplicity in the duality of thinking and what is thought – the *Nous* is even seen as the totality of all ideas. The *Nous* does not display an absolute Unity, for it exhibits a balance between unity and multiplicity, as unity-in-the-multiplicity (*hen polla*, cf. *En*. VI,2,2,2; VI,7,14,11-12). However, a "Unity-Multiplicity," owing to its inherent multiplicity is *evil* compared to the pure unity of the One (*En*. VI,6,3). In this sense, in contradiction with *En*. I,8,3, evil does belong to the sphere of being of the *Nous*. In sensory things, composed of *form* and (evil) *matter*, multiplicity even dominates (cf. *En*. 111,2,2). Mühlenberg emphasizes that for Plotinus *infinity* is essentially *evil* and lacks any determination in itself (Mühlenberg 1966:82, 85 ff.).

11. INFINITY: SIMPLICITY, ENDLESSNESS, INDIVISIBILITY, IMMUTABILITY AND ETERNITY

Aristotle already discerned a connection between *immutability* and *eternity* (*De coelo* A 10, 279b21-24). Clement (150-215) employs the term *apeiron* in his explanation of God's infinity. Osborn translates his view as follows: "Therefore it is infinite, not merely in the sense that one cannot give an exhaustive account of it, but in the sense that one cannot analyse it into parts and that it has no limit and is therefore without form or name" (Osborn 1957:29). Mühlenberg questions the insertion of "merely" in the translation of Osborn and points out that what Clement has in mind is the absolute *simplicity* and *indivisibility* of God's *infinity*. He here distinguishes between two features: (a) infinity in the sense of *not coming to an end* and (b) infinity related to God understood as *One* who cannot be composed out of an infinite number of parts (see Mühlenberg 1966:75).

12. THE TRANSITION FROM EVIL INFINITY TO A POSITIVE APPRECIATION

After Anaximander assigned a divine nature to the *apeiron* Greek philosophy witnessed the shift from the *matter pole* to the *form pole*. This generated a negative appreciation of the *apeiron*. It was Anaxagoras and after him Plato and Aristotle who depreciated matter and the infinite inherent to it. Eventually it was Plotinus who once more introduced a connection between infinity and what is considered to be *divine*, namely the *One*. The remarkable fact is that Plotinus characterizes both the One (out of which everything arises) and the contrasting matter, the permanent substrate, as *infinite* (cf. *En*. II,4,4; II,4,10; II,4,15; VI,7,32) Although the term *infinite* is therefore employed in a dialectically opposed manner with regard to the One and (formless) matter, it still merely receives form (as a permanent substratum) while the (formless) One *gives form* (cf. *En*. VI,7,17). In addition, as noted earlier, this re-appreciation is related to Plotinus' view of *eternity* implicitly equated with *infinity* as the *timeless present* (cf. the whole *En*.III,7). This simultaneously exerted a considerable influence on the conceptions of infinity found in later thinkers, such as Boethius, Augustine (*Confessiones* XI,11,13; *De Trinitate* XII,14), Thomas Aquinas (*Summa Theologica* I,10) and Schilder (1948:61).

Gregory of Nyssa used Plato and Aristotle as a stepping stone to advance his own positive appreciation of the infinite – and in doing that at once exceeded their views. Gregory points out that Aristotelian logic precludes assigning the property of infinity to God (Mühlenberg 1966:180). Therefore, he provides a foundation for his elimination of Aristotle's logic through ascending in endlessness to the infinity of God (Mühlenberg 1966:165). We have to note the subtle distinction between *endlessness* and *infinity*.

Gregory of Nyssa proceeds on the basis of something immutable, a pure being and absolute perfection (Mühlenberg 1966:120, 123). The systemic part of the work of Gregory of Nyssa on "Contra Eunomium," commences with explaining that Eunomius pursue God as his first principle, from which it follows that it is "the highest and most authentic being." For Eunomius God is at the peak of a hierarchically branching pyramid. Augustine already designated God as the highest being (*ipsum esse*). The way in which Eunomius expresses himself eventually

became a widespread element of theological and philosophical parlance – just compare the work of Lovejoy (1960) on "*The Great Chain of Being*."

Gregory of Nyssa incorporates ancient Greek views by holding that God is *simple* and *immutable* (Mühlenberg 1966:124). But when he understands limitlessness at once as perfection the road is opened to use infinity positively as the essence of God (see Mühlenberg 1966:125-126). At the same time it leaves room for distinguishing between "endlessness" and "infinity" – something already accomplished by Plotinus.

13.   ENLARGING THE SCOPE OF ENDLESSNESS AND INFINITY

Mühlenberg repeatedly points out that Gregory of Nyssa considers infinity to be the *essence* of God. Theology presupposes this revelation of infinity as the essence of God (see Mühlenberg 1966:195). Gregory of Nyssa recognizes God's transcendence in His infinity, but at the same time he holds that God does not exist within the domain of time, for time forms part of God's creation (Mühlenberg 1966:135).

Augustine further explored the view of Plotinus by explicitly asserting that our inability to understand the infinite should not be used as a measure for God, since in his omniscience God understands every infinity – also the completed infinite sequence of all numbers – without any passage of thought, at once, without before and after. Therefore, God can also know his own completed infinite being. Creation, however, is *finite*. Plotinus speaks of "everything together" enabling observing the unified infinity at once (*En*. 5,10,7 ff.). These terms intimately relate to key features of the actual infinite, to which we shall return presently.

Georg Cantor calls upon Augustine in justifying his exploration of the domain of the *actual infinite*. He points out that Augustine considers the intuitive perception of the succession of natural numbers and contrast it with intuiting all these individuals at once as a whole or totality. According to Augustine God is capable to know these numbers "*formaliter* as an actually-infinite whole, as a transfinite" ["*formaliter* als ein aktuell-unendlichen Ganzes, als ein transfinitum an"] (Cantor 1962:402). At this point we enter the early medieval wrestling with what Maier later on highlighted, in connection with God's infinity, as the contest between the *successive infinite* (*infinitum successivum*) and the *at once infinite* (*infinitum simultaneum*) (see Maier, 1964:77-79 and Maier 1949). These two expressions, the *successive infinite* and the *at once infinite*, have a clear intuitive meaning, something lacking in the traditional designations, the *potential infinite* and the *actual infinite*. Phrased in general terms, one can say that any given a *successively infinite* sequence of numbers could always, by employing the *successive infinite*, be viewed *as if* they are all given *at once* as an infinite whole or totality.

Only by the end of the middle ages and the beginning of the modern era do we observe a change in this view when Cusanus altered this understanding with his doctrine that God is actually infinite while reality is only endless. Linked to his conviction that the infinite line is simultaneously a triangle, circle and sphere (*De Docta Ignorantia*, I,13-17 – see Hopkins 1985:20-23), Cusanus taught that of God, as the actually infinite, one could in a certain sense say everything and nothing at all (he is e.g. the biggest and the smallest – *De Docta Ignorantia*,

I,5) since all contradictions are resolved in him (*coincidentia oppositorum*) (*De Docta Ignorantia*, I,22; *De Coniecturis* II,1 and II,2). According to K. Kremer Plotinus and Proclus actually already taught that all contradictions are resolved in God (Kremer 1966:354).

Descartes explores the increasing positive appreciation of the actual infinite, the *at once infinite*. According to him the "extension of the world is indefinite" (Principles XXI). He turns the classical conception upside down by viewing the infinite as *complete* and the finite as *incomplete*. Therefore, the finite should rather be seen as the *non-infinite*. Spinoza not only identified God with nature (*Deus sive natura*), but also saw the universe as completed infinite.

This legacy is still present in the thought of Maimon (1753-1800) who distinguishes between "an absolute [understanding] (not limited by sensibility and its laws)" and "our understanding, according to its delimitation" (Maimon 1790:227). Maimon considers the complete sequence of all natural numbers and concludes that it is not an object that could be given in our intuition. It is rather a mere idea through which the successive progression into infinity is viewed as an object. But reason here contradicts itself "insofar as it views something as an object that according to its conditions can never be seen as an object" (Maimon 1790:228). According to Maimon the solution should be explained as follows: an infinite number (because our perception is bound to the form of time), cannot be represented other than as an *infinite succession in time* (which consequently is not capable of being completed). In the case of an absolute understanding, by contrast, the concept of an infinite number is *thought of at once*, without any passage of time. (This distinction is the equivalent of our distinction between the *successive infinite* and the *at once infinite*.)

14. ENDLESSNESS AND TOTALITY

Interestingly Wild mentions that Levinas holds the view that our basic philosophical questions transcend metaphysics and ethics – my "way of existing conveys my final answer." He points out that Levinas distinguishes between "totalizers" and "infinitizers":

> As Levinas points out, one answer is given by the totalizers who are satisfied with themselves and with the systems they can organize around themselves as they already are. A very different answer is given by those who are dissatisfied, and who strive for what is other than themselves, the infinitizers, as we may call them. The former seek for power and control; the latter for a higher quality of life. The former strive for order and system; the latter for freedom and creative advance. This leads to the basic contrast which is expressed in the title of the book, between totality on the one hand and infinity on the other. Many examples of the former can be found in the history of our Western thought. The latter is largely unknown and untried.
>
> It is this outwardly directed but self-centered totalistic thinking that organizes men and things into power systems, and gives us control over nature and other people. Hence it has dominated the course of human history" (Wild 1979:17).

Views such as what we have discussed thus far cannot bypass the developments of 19th century mathematics since it was during this period that the actual infinite was employed in key

mathematical discoveries for the first time – exploring the foundation provided by Bolzano in his earlier mentioned *Paradoxien des Unendlichen*.

The three mathematicians who subsequently worked on the foundations of mathematics during the last three decades of the 19th century are Weierstrass, Dedekind and Cantor. The latter explains his understanding of the potential infinite and actual infinite (in our terminology: the *successive infinite* and the *at once infinite*), as follows:

> In general I refer to the potential infinite wherever an undetermined magnitude comes into view, which is capable of uncountable many determinations. Under an actually infinite, in contrast, is a *Quantum* to be comprehended that on the one hand is *not variable*, but much rather in all its parts firm and determined, a genuine constant, while at once on the other exceeding every finite magnitude of the same kind in size (Cantor 1962:401).

About 50 years later, when Kowalewski wrote his memoirs, he often refers to the achievements of Cantor. Meschkoski reports what Kowalewski said about the "gapless ascend" of cardinalities in the series of Alephs:

> These cardinalities [Mächtigkeiten], the Cantorean Alephs, were for Cantor something sacred, certainly steps which reach out to the throne of God as the absolutely infinite (cf. Meschkowski 1967:111). He was convinced that with these Alephs all conceivable cardinalities were fulfilled. [11]

15.   INFINITY BELONGS TO MATHEMATICS IN THE FIRST PLACE

While Cantor therefore brought both the *successive infinite* and the *at once infinite* back home according to their mathematical meaning, he had to acknowledge God's infinity as something transcending mathematics with its two kinds of infinity. We now have the *successive infinite*, the *at once infinite*, and the *absolute infinite* (God).

In 1899 Cantor wrote in a letter to Dedekind that a multiplicity (Vielheit) may be structured in such a way that the assumption of the "being together" of *all* its elements leads to a contradiction. Multiplicities such as these are designated by him as "inconsistent multiplicities" or as "absolute multiplicities" (see Cantor 1962:443).

Hermann Weyl, the gifted student of Hilbert, also wrestled with these issues. But in his case his orientation was influenced by the intuitionism of Kronecker and Brouwer – an alternative trend in mathematics rejecting the actual infinite (*at once infinite*). His position reflects the transition from Greek philosophy up to the convictions of Augustine. We have noted that Augustine believed that our inability to understand the infinite should not be used as a measure

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[11]   "Diese Mächtigkeiten, die Cantorschen Alephs, waren für Cantor etwas Heiliges, gewisserma en die Stufen, die zum Throne der Unendlichkeit, zum Throne Gottes emporführen. Seiner Oberzeugung nach waren mit diesen Alephs alle überhaupt denkbaren Mächtigkeiten erschopft" (Meschkowski 1967:110, 154)

for God, since in his omniscience God understands every infinity – also the completed infinite sequence of all numbers – without any passage of thought, at once, without before and after.

The basic conception of Weyl is quite similar to what Augustine held. He accepts only the *successive infinite* as mathematically sound and therefore rejects the use of the *at once infinite* in mathematics.

> The infinite is accessible to the mind intuitively in the form of the field of possibilities open into infinity, analogous to the sequence of numbers which can be continued indefinitely … but the completed, the actual infinite as a closed realm of absolute existence is not within its reach (Weyl 1932:83).

He holds that mind is "open toward the infinite," but "God as the completed infinite cannot and will not be comprehended by it" (Weyl 1932:84). This view approximates the view of Gregory of Nyssa for we have seen that the latter holds that "infinity" is the "essence of God."

From the perspective of modern physics Einstein proposed, in his work on relativity (containing both the special and general theories of relativity), that we may see the universe as "finite" and yet "unbounded" (see Einstein 1920:108-112). While Cantor still believed that the (physical) universe is (actually) infinite, contemporary mathematics and physics settled for a *finite universe*. Penrose remarks that he assumes "that we live in a 10^{80} baryon universe" (see Penrose 2004:729).

Notions of eternity are linked to the classical understanding of the *present* in which everything coheres (see the view of Parmenides, Diels-Kranz B Fr.8:3-6). Beierwaltes reflects on this legacy in relation to the *Enneads* III:7 of Plotinus. Via Boethius it entered the modern scene up to Kierkegaard's *nunc aeternum* (the eternal now).

With reference to the New Testament conception of *kronos* Cullmann is convinced that it should not be understood in the sense of "timelessness" (Cullmann 1949:49). According to him the "New Testament makes absolutely no difference in terminology. Eternity is the endless succession of the ages" (Cullmann 1949:62).

Cullmann therefore defends a conception of eternity in terms of "successive ages" while rejecting the view of eternity as timelessness. What he does not realize is that *behind* theological notions of eternity two alternative views of infinity are found, namely the *successive infinite* and the *at once infinite*. The former suggests an endless duration of time and the latter timelessness, connected to the *present* (and spatial *simultaneity*). Consider for a moment how Wittgenstein, in his *Tractatus Logico-Philosophicus*, continues to explore the Parmenidean, Platonian, Plotinian and Boethiusian legacy of eternity as timelessness, manifested in the present:

> If we take eternity to mean not infinite temporal duration but timelessness, then eternal life belongs to those who live in the present (6.4311).

A sincere understanding of infinity cannot avoid giving an account of the meaning of the two kinds of infinity that governed the development of mathematics. First of all, the *successive infinite* brings to expression the primitive numerical meaning of succession and the idea of

being given at once as an infinite totality makes an appeal to the irreducible spatial whole-parts relation.

Bertrand Russell is justified in his claim that the "relation of whole and part is, it would seem, an indefinable and ultimate relation" (Russell, 1956:138). He also realized that the relation of "greater and less" is "undefinable." This entails that he implicitly accepts the primitive meaning of *numerical succession* (see Russell, 1956:194). Subsequently he remarks that "progressions are the very essence of discreteness" (Russell, 1956:299) and therefore, later on, criticizes Bolzano for not distinguishing the "many from the whole which they form" (Russell, 1956:70). This *whole*(ness) forms an ultimate building block of Cantor's concept of a set for he claims that

> Under a 'set' we understand every collection *M* of specific properly distinguished objects *m* of our intuition or of our thinking (which are called 'elements' of *M*) into a whole [Unter einer 'Menge' verstehen wir jede Zusammenfassung *M* von bestimmten wohlunterschiedenen Objekten *m* unsrer Anschauung oder unseres Denkens (welche die 'Elemente' von *M* genannt werden) zu einem Ganzen (Cantor 1962:282)].

Clearly, the idea of *infinite totalities* underlying Cantor's transfinite arithmetic presupposes an awareness both of succession and of the whole-parts relation. Since the latter is basic to our spatial intuition it is impossible to provide a foundation for the *at once infinite* apart from the intimate connection between number and space. It is therefore all the more surprising that Levinas wrote a whole book on "Totality and Infinity" without reflecting on the mathematical meaning of "infinite totalities"!

16.   CONCLUDING REMARK

The initial understanding of infinity in Greek philosophy commenced by viewing it as divine (the *arch* or *origin*). Anaximander designated it as the *apeiron* (the infinite-unlimited). After the form motive acquired primacy in Greek culture infinity, closely associated with formless matter, was increasingly de-divinized, explaining why infinity as attribute was not predicated of God. However, the idea of the actual infinite (the *at once infinite*), eventually helped Plotinus, Gregory of Nyssa and Augustine to argue that what is unlimited and infinite could be observed by God in one glance, at once as an infinite totality present without before and after.

It is therefore not surprising that Cantor made an appeal to Augustine in order to help justifying his mathematical theory of transfinite numbers. Yet, after Cantor introduced his transfinite arithmetic the discovery of the antinomies of set theory inspired the neo-intuitionism of Brouwer, Weyl, Heyting and others to reject the *at once infinite*. In his article on *Mathematics and Logic* (1946) Weyl understands infinity "in the precise sense that the members of such a set are explicitly exhibited one by one." He proceeds:

> Forgetful of this limited origin, one afterwards mistook that logic for something as above and prior to all mathematics, and finally applied it, without justification, to the mathematics of infinite sets. This is the Fall and original sin of set-theory, for which it is justly punished by the antinomies" (Weyl 1946:10).

The axiomatization of set theory only partially saved the day, because in 1931 Gödel has shown that axiomatic consistency spells incompleteness Grünfeld remarks:

> Gödel proved that if any formal theory T that is adequate to include the theory of whole numbers is consistent, then T is incomplete. This means that there is a meaningful statement of number theory S, such that neither S nor not-S is provable within the theory. Now either S or not-S is true; there is then a true statement of number theory which is not provable and so not decidable. The price of consistency is incompleteness (Grünfeld 1983:45).

Given the key role of infinity in mathematics Weyl is certainly justified in saying that "[M]athematics is the science of the infinite" (Weyl 1932:7). Nonetheless the implication of Gödel's 1931 proof was that every axiomatic system proceeds from an intuitive insight transcending the formalism of the system. Weyl strikingly comments: "It must have been hard on Hilbert, the axiomatist, to acknowledge that the insight of consistency is rather to be attained by intuitive reasoning which is based on evidence and not on axioms" (quoted in Reid 1970:269). [12]

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[12]   The fact that infinity is not a purely arithmetical term is explained from a different angle in a recent article published in PONTE (see Strauss 2018).

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